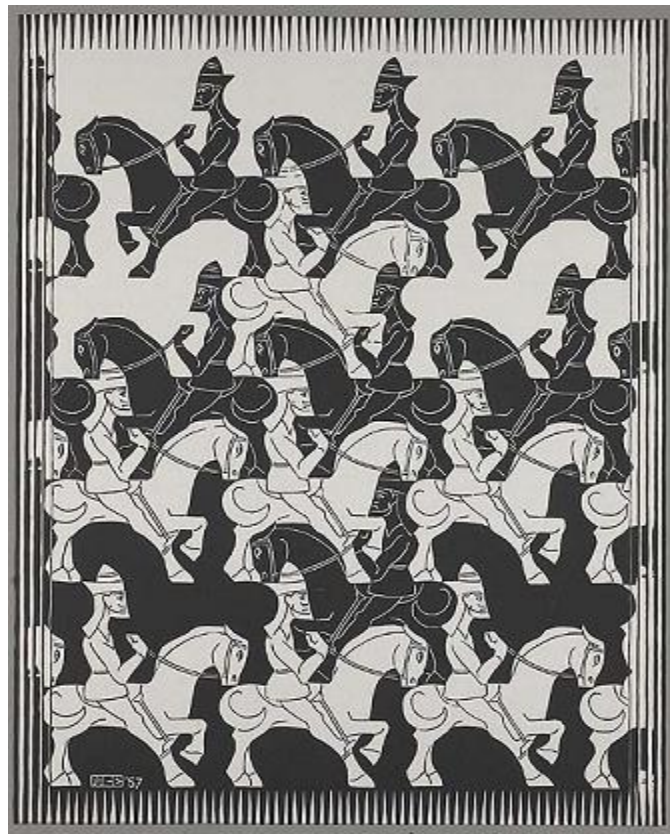


### **Biography of İlhan M. İzmirli, Ph.D.**

I grew up surrounded by books and a profound sense of deference for education, a practice, which, according to Oscar Wilde (1854 – 1900), “makes one rogue cleverer than another,” which, I interpreted as the pursuit of knowledge at once esoteric and abstract for the sole purpose of attaining intellectual maturity, cultural sensitivity, and social consciousness. I was equally attracted to literature, mathematics, music, and physics and was having a hard time deciding which one of these disciplines to study in college. It was when I was, as a high school student, exposed to the indubitably enthralling facts that serious mathematics existed naturally in the universe and that it arose from relatively simple situations and reached immense levels of abstraction, that I knew that I was going to study mathematics.

My first encounter with the astounding prevalence of mathematics in nature had something to do with the *tessellation* (from the Greek *tessera* meaning square, derived in turn from the word for four) of the plane, that is, a covering of the plane with figures that fill the plane with no overlaps and no gaps. For stunningly ingenious examples of plane tessellations, one should see the paintings of the Dutch graphic artist Maurits Cornelis (M. C.) Escher (1890 – 1972)



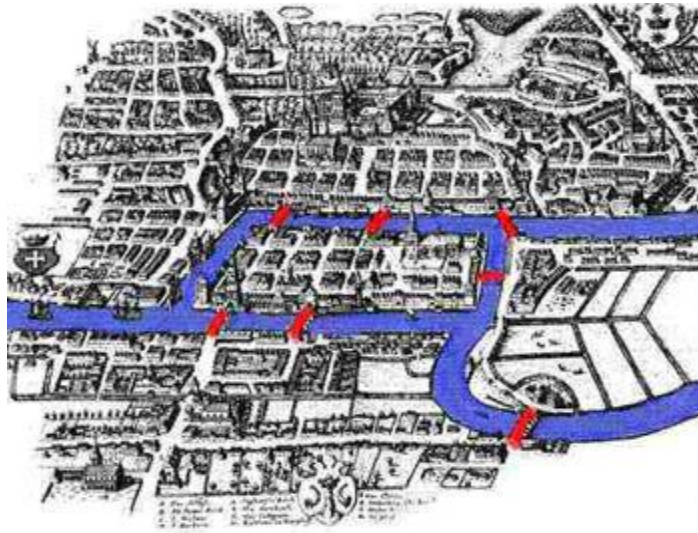
Regular Division of the Plane III by Escher

([http://en.wikipedia.org/wiki/File:Escher,\\_Regular\\_Division\\_of\\_the\\_Plane\\_III.jpg](http://en.wikipedia.org/wiki/File:Escher,_Regular_Division_of_the_Plane_III.jpg))

A *regular tessellation* is a tessellation made up of congruent regular polygons. It is easy to show that only three regular tessellations exist: those made up of equilateral triangles, squares, and hexagons.

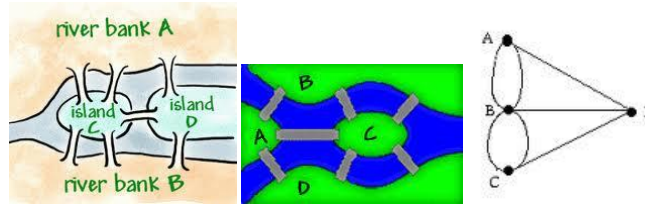
Now which one of these tessellations is the most efficient one, that is, for a given fixed perimeter, which one of the above figures encloses the maximum area? The answer turns out to be the hexagon: if a fixed number of  $x$  units is given as the perimeter, the area of the equilateral triangle will be  $\frac{\sqrt{3}}{36}x^2$ , the area of the square will be  $\frac{1}{16}x^2$ , and the area of the regular hexagon will be  $\frac{\sqrt{3}}{24}x^2$ . Hence, roughly speaking, if we are given 1 unit of material to form the perimeter, the triangle will cover an area of 0.0481 square units, the square 0.0625 square units, and the hexagon 0.0722 square units. Thus, when bees form hexagonal cells to build their hives, they are using the minimum amount of material to enclose maximum amount of area.

As for some simple problems leading into serious mathematics, one such problem I was exposed to was the famous *Bridges of Königsberg* problem and the Swiss mathematician Leonhard Euler's (1707-1783) brilliant solution of it. The city of Königsberg (now Kaliningrad) was set on both sides of the Pregel River with two islands in the middle. The two sides of the town and the two islands on the river were connected to each other by seven bridges. There was, in the local culture, a perplexingly simple query that the townspeople had not been able to answer for centuries: is it possible to walk through the city by crossing each bridge once and only once? This problem became known to Euler, who had stopped in the town on his way to his new teaching post in St. Petersburg, Russia, from his native Basel. Before long, Euler proved that the answer was "no".



The Bridges of Königsberg (Königsberg\_bridges.png)

He did so by formulating the question in an abstract setting (hence laying the foundations of modern graph theory), and by showing that this walk would have been possible if and only if each *vertex* (land mass) had an even order, in other words, if and only if each vertex was connected to each other vertex by an even number of *edges* (bridges) as depicted below:



Euler's imposing and exciting solution was published as *Solutio problematis ad geometriam situs pertinentis* (The solution of a problem relating to the geometry of position) in 1741.

The French/Algerian author and philosopher Albert Camus (1913-1960) once wrote "A man's work is nothing but this slow trek to rediscover, through the detours of art, those two or three great and simple images in whose presence his heart first opened." Indeed, this picture remained with me throughout my college years.

Another such remarkable problem was the one that inspired the development of mathematical probability in Renaissance Europe. Two players, both of whom start a game with an equal chance to win and the same amount of money,  $x$ , are interrupted while playing the game. Given the score of the game at that point, how should the total amount of  $2x$  be divided?

This ostensibly mundane and undemanding question started a long correspondence between two prominent French mathematicians of the period, Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665) that heralded the beginnings of mathematical probability. Roughly, here was the ensuing argument.

Suppose the game is one of flipping coins. If the outcome is heads (H), player A gets a point, and if it is tails (T), player B gets a point. The first player to get 10 points wins the game, and the winner takes all. At some stage of the game, when A was ahead 8 points to 7, the game was unavoidably interrupted, and one of the players had to leave right away. How should the stakes be divided?

Since A needed only two points to win the game and B three, after four more tosses of the coin, the game would have been over, for there can be no combination of four outcomes that does not contain either two H's or three T's, which can easily be shown by observing the 16 such possible outcomes:

HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, ... , TTTT

Assuming all these outcomes are *equally likely*, since there are 11 cases where two H's precede three T's (the only ways B can win are TTTT, TTTH, TTHT, THTT, and HTTT), the ratio of A's chance of winning to B's chance of winning is 11: 5 and the total amount of money on the table,  $2x$ , should be divided in that ratio, giving A  $\frac{11}{16} (2x) = \frac{11x}{8}$  and B  $\frac{5}{16} (2x) = \frac{5x}{8}$ . So, for example, if each player started the game with 40 dollars, A would end up with 55 dollars and B with 25.

I studied mathematics at Bosphorus University, Istanbul, with a minor in physics. Then I started working on my M.S. at the University of Istanbul. At the time, having read *A Mathematician's Apology*, a 1940 essay by the famous English mathematician G. H. Hardy (1877-1947) - possibly the best apology ever written short of the Socratic one - I had acquired a predilection for abstract theoretical mathematics, especially, number theory. So for my M.S. thesis, I worked on a remarkable problem in number theory.

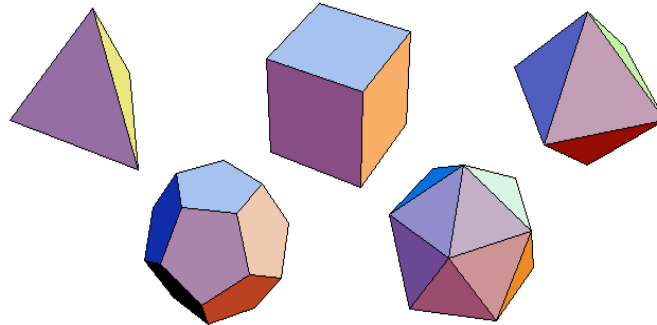
In 1770, the English mathematician Edward Waring (1736-1798) came up with the following conjecture: For every natural number  $k$  there exists an associated natural number  $m$  such that every natural number  $r$  can be written as the sum of at most  $m$   $k^{th}$  powers of natural numbers, that is,

$$r = x_1^k + x_2^k + \dots + x_m^k$$

where  $x_1, x_2, \dots, x_m$  are natural numbers and  $k$  and  $m$  are independent of  $r$ . This claim got to be known as the *Waring Conjecture*.

Some individual cases were proved early on. For example, the French/Italian mathematician Joseph Louis Lagrange (1736-1813) showed in 1770 that every natural number could be written as the sum of 4 squares (*Lagrange's four-square theorem*). Later it was shown that any natural number could be written as the sum of 9 cubes and 19 fourth powers. The general case was proved by the German mathematician David Hilbert (1862-1943) in 1909. Then on, this result was referred to as the *Hilbert–Waring Theorem*. Hilbert's proof was heavily dependent on complex analysis. My goal was to simplify Hilbert's "complex" approach.

I completed further graduate studies at the University of South Carolina, Columbia, S.C. Meanwhile, my fascination in surprisingly elegant and inherently indispensable relationships between theoretical mathematics and applied sciences - the astonishing utility and splendor of mathematics - kept on growing. For instance, how could one fail to be impressed by the use of regular polyhedra (the Platonic solids) in Keplerian cosmology?



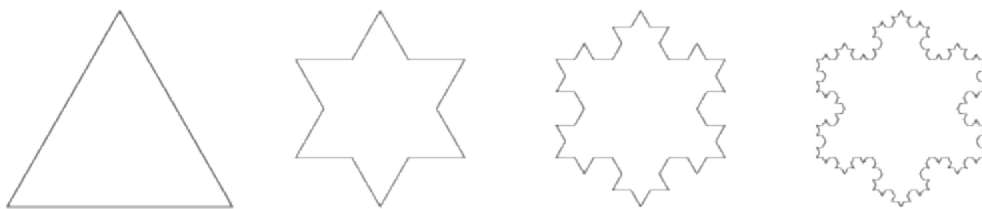
The Five Platonic Solids

In his famous book *Mysterium Cosmographicum*, published in 1596, the German mathematician and astronomer Johannes Kepler (1571-1630) sought a relation between the five planets known at that time besides the Earth (Mercury, Venus, Mars, Jupiter, Saturn) and the five Platonic solids. The solids were ordered with the innermost being the octahedron, followed by the icosahedron, the dodecahedron, the tetrahedron, and finally the cube. In this way the structure of the solar system and the distance relationships between the planets was dictated by the Platonic solids.

Eventually, this idea had to be discarded, but out of this exploration came the deduction that the orbits of the planets were ellipses rather than circles.

Or, how could one not be astounded by the use of fractal geometry to measure the coastline of England? Fractals, of course, (the term *fractal* was coined by Benoît Mandelbrot (1924 - ) in 1975 from the Latin *fractus* meaning broken) are geometric objects too irregular to be described in traditional Euclidean geometric terminology. They are much better suited than regular geometric figures to depict entities commonly found in nature, such as clouds, snowflakes, and coastlines, as well as blood and pulmonary vessels.

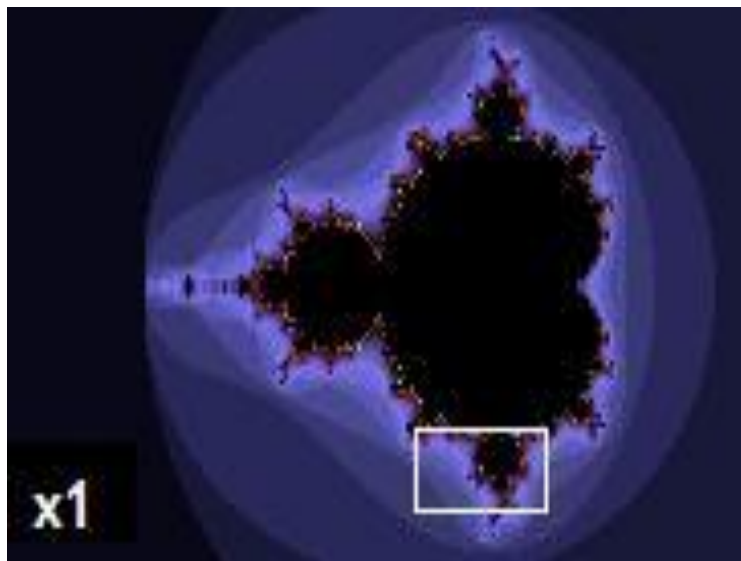
Fractals have several interesting properties. One such is the property of *self-similarity*: a fractal can be split into smaller and smaller parts, each of which is similar or approximately similar to the original. For example, starting with an equilateral triangle, removing the middle third of each side and building an equilateral triangle at that location, we get a fractal called the *Koch snowflake*, described by the Swedish mathematician Helge von Koch (1870-1924) in 1904:



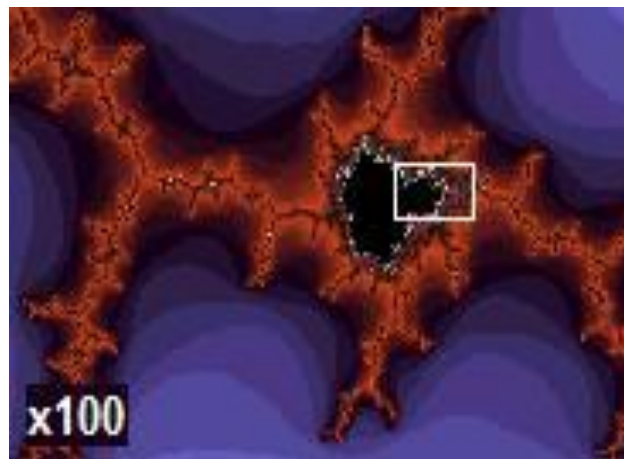
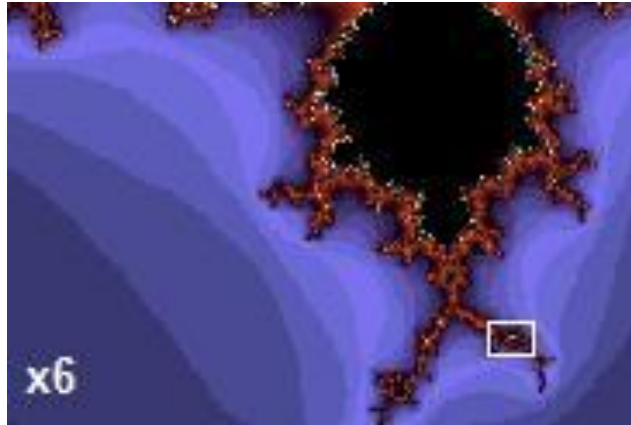
Koch Snowflake (<http://mathworld.wolfram.com/Kochsnowflake.html>)

Another interesting property is that *fractal dimensions* (*Hausdorff dimension*) are not natural numbers. Here are pictures of some well-known fractals:

#### 1. The Mandelbrot Set

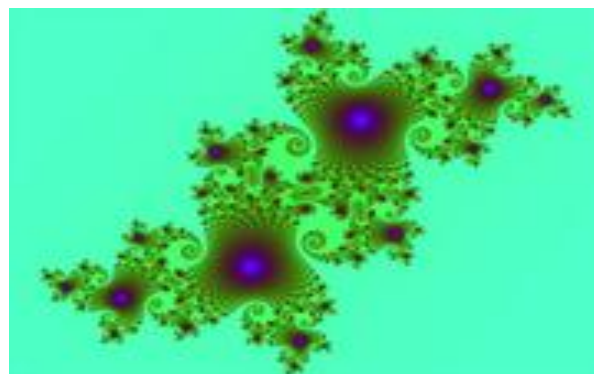






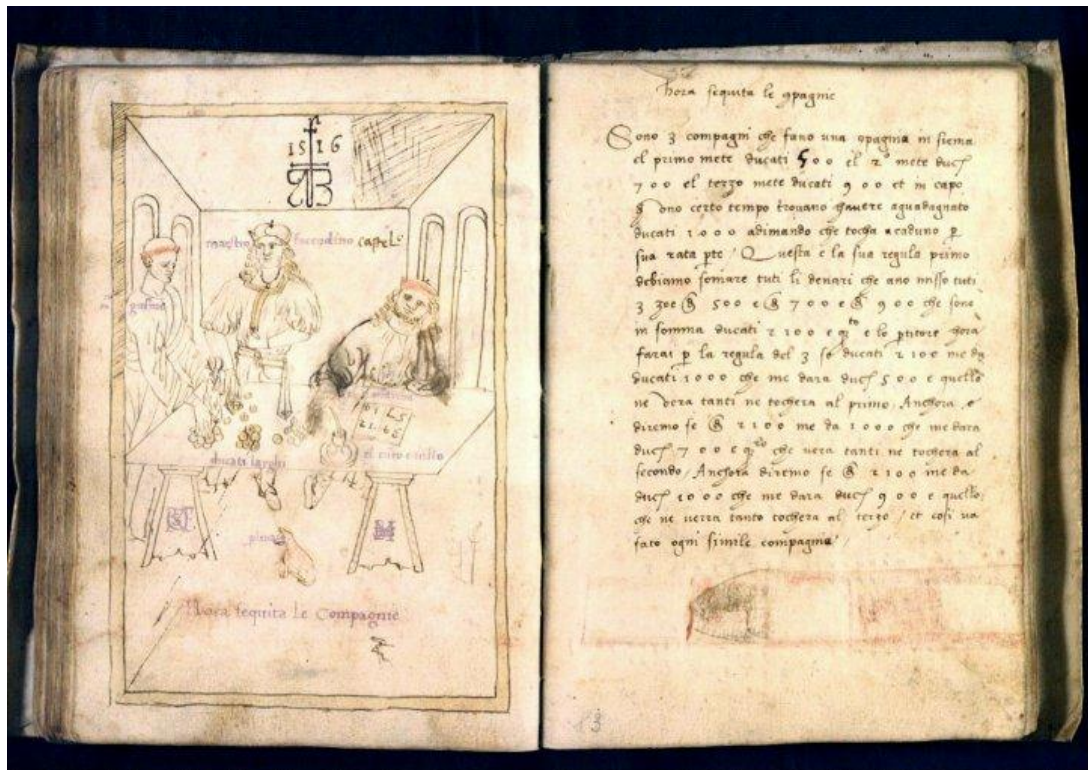
The Mandelbrot set and its magnifications ([http://en.wikipedia.org/wiki/File:mandelbrot\\_set](http://en.wikipedia.org/wiki/File:mandelbrot_set))

## 2. The Julia Set



A Julia Set is a fractal related to the Mandelbrot set ([http://en.wikipedia.org/wiki/File:julia\\_set](http://en.wikipedia.org/wiki/File:julia_set)). It was introduced into mathematical literature by the French mathematician Gaston Julia (1893-1978)

Or take the Fibonacci numbers (Fibonacci (son of Bonacci) was the nickname for the Italian mathematician Leonardo da Pisa (c. 1170 – c. 1250)). In his 1202 book *Liber Abaci*, Fibonacci posed the following question: Suppose at the beginning of the year, we have a newly-born pair of rabbits, a male and a female. Assume moreover, that these rabbits do not die for the year and that they always produce one new pair (one male, one female) every month from the second month on. How many pairs will there be at the end of the year?



A page from *Liber Abaci* (<http://www.comune.genova.it/servlets/resources?resourceId=701894>)

Clearly, at the end of the first month, there would still be only one pair, at the end of the second month there would be two pairs, and so on, giving the sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

If we let the number in the  $k^{th}$  month be denoted by  $f_k$ , the solution can, obviously, be written in terms of the recursive definition

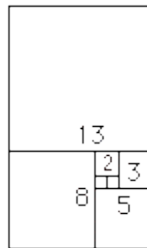
$$f_1 = 1, f_2 = 1, f_{n+1} = f_n + f_{n-1}$$

Generalizing the above formula, we get the infinite sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, ...

This is the sequence referred to as the *Fibonacci sequence*. On many plants, the number of petals is a Fibonacci number: buttercups have 5 petals; daisies can be found with 34, 55, and 89 petals.

Here is a geometric portrayal of the Fibonacci numbers. Start with two small squares of size 1 next to each other. On top of both of these draw a square of size 2 (= 1+1). Now draw a new square - touching both one of the unit squares and the square of side 2 - thus having sides 3 units long; and so on. The resulting set of rectangles will have sides that are two successive Fibonacci numbers in length.



Now if we construct an approximate spiral (*logarithmic spiral*) drawn in the squares, we will get the following figure



which is the shape of the spirals observed on the shells of snails, sea shells, pine cones, and in the arrangement of seeds on flowering plants.

Indeed this curve fascinated mathematicians throughout centuries. Jakob Bernoulli referred to it as the *spira mirabilis*, the miraculous spiral. It has the peculiar property that its size increases but its shape remains unaltered. It is possibly because of this property that it appears in the growth pattern of certain natural objects.

The equation of this rather complicated curve can be written parametrically as

$$\begin{aligned} x(t) &= ce^{kt} \cos t \\ y(t) &= ce^{kt} \sin t \end{aligned}$$

where  $c$  and  $k$  are two real constants.

Closely related to the Fibonacci numbers are the *Lucas numbers*, introduced by the French mathematician François Édouard Anatole Lucas (1842-1891). If instead of taking 1 and 1 as our initial



values, we take two arbitrary numbers, say  $a$  and  $b$ , we get a sequence of numbers known as the *Lucas numbers*,  $g_n$  defined by the recurrence relation

$$g_1 = a, g_2 = b, g_{n+1} = g_n + g_{n-1}$$

yielding the infinite sequence

$$a, b, a + b, a + 2b, 2a + 3b, 3a + 5b, \dots,$$

It is easy to see that

$$g_n = af_{n-1} + bf_n$$

There is yet another interesting property of the Fibonacci numbers. If we take the ratio of two successive numbers in this series, we get

$$\frac{1}{1} = 1, \quad \frac{2}{1} = 2, \quad \frac{3}{2} = 1.5, \quad \frac{5}{3} = 1.666 \dots, \quad \frac{8}{5} = 1.6, \quad \frac{13}{8} = 1.625, \quad \frac{21}{13} = 1.61538 \dots,$$

$$\frac{34}{21} = 1.61904 \dots, \quad \frac{55}{34} = 1.61764 \dots, \quad \frac{89}{55} = 1.618181 \dots, \quad \frac{144}{89} = 1.61797 \dots$$

It can be shown that

$$\lim_{n \rightarrow \infty} \frac{f_{n+1}}{f_n} = \frac{1+\sqrt{5}}{2} = 1.618034 \dots = \Phi.$$

In fact, in 1842, French mathematician Jacques Philippe Marie Binet (1786-1856) proved that

$$f_n = \frac{\Phi^n - (-\Phi)^n}{\sqrt{5}}$$

This number  $\Phi$  is called the *golden ratio* (from the Latin *aurea section*, the golden section) or the *divine ratio* (from the Latin *sectio divina*, the divine section). It arises naturally in some partition problems. For instance, suppose a rod of length  $w$  is to be divided into two unequal parts in as much pleasant a way as possible. This would entail the ratio of the shorter part to the longer part to be equal to the ratio of the longer part to the whole. So, if  $s$  denotes the shorter part and  $l$  the longer part, we should have

$$\frac{s}{l} = \frac{l}{w} = \frac{l}{l+s}$$

giving us the quadratic equation,  $l^2 - ls - s^2 = 0$ . But this implies  $\frac{l}{s} = \frac{1 \pm \sqrt{5}}{2}$ , and since the ratio of lengths cannot be negative, we must have

$$\frac{l}{s} = \frac{1 + \sqrt{5}}{2} = \Phi$$

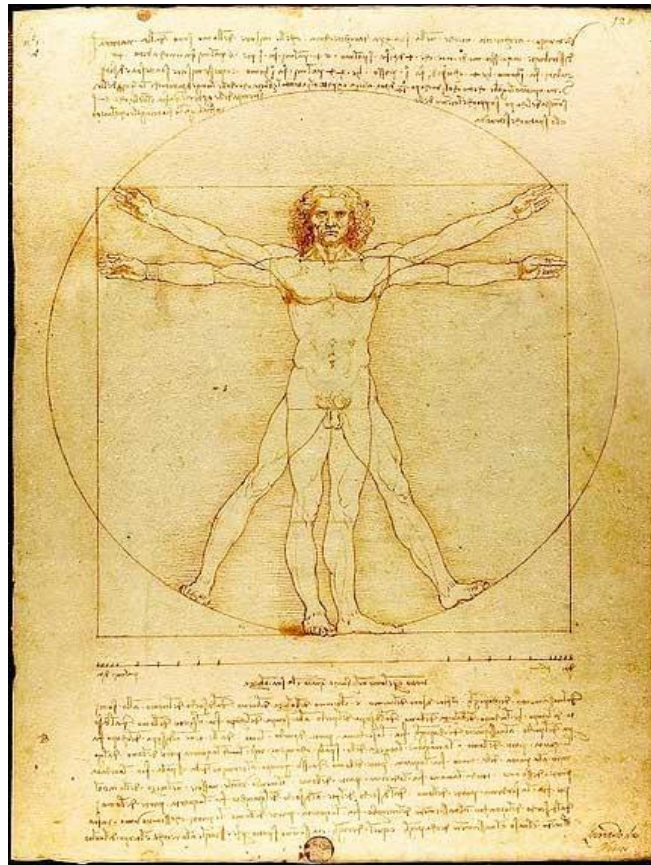
It clearly follows that

$$\frac{1}{\Phi} = \Phi + 1$$

It is also easy to see that  $\Phi$  is irrational. For if  $\Phi$  were rational, then by closure properties of the rational numbers,  $2\Phi - 1 = \sqrt{5}$  would be rational, a contradiction.

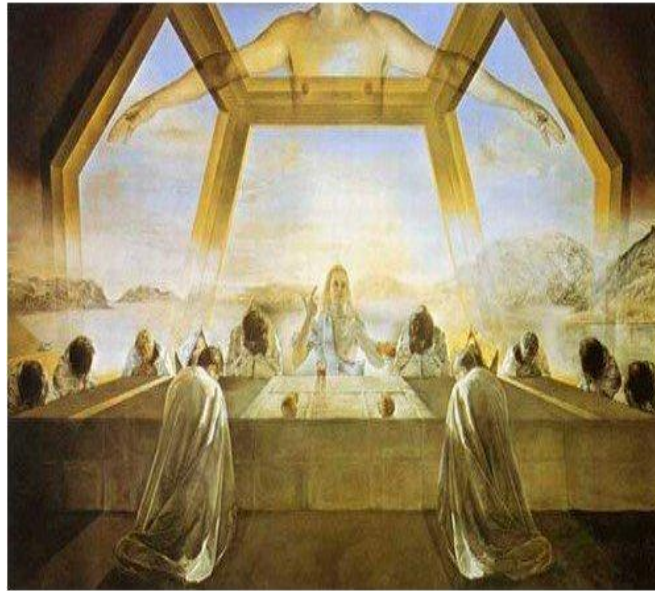
One of the earliest works on the golden ratio was the *De Divina Proportione* by the Italian mathematician and Franciscan friar Luca Pacioli (1446-1517), a three-volume work published in 1509. What makes this book so historically valuable is that the illustrations of regular solids in it were made by none other than Leonardo da Vinci (1452-1519), Pacioli's longtime friend and collaborator.

The golden ratio has been used extensively in the arts. For example, in da Vinci's 1487 drawing, *Vitruvian Man*, where a nude male figure is depicted in two superimposed positions inscribed in a circle and square, and the proportions of the circle and square reflect, approximately, the golden ratio.



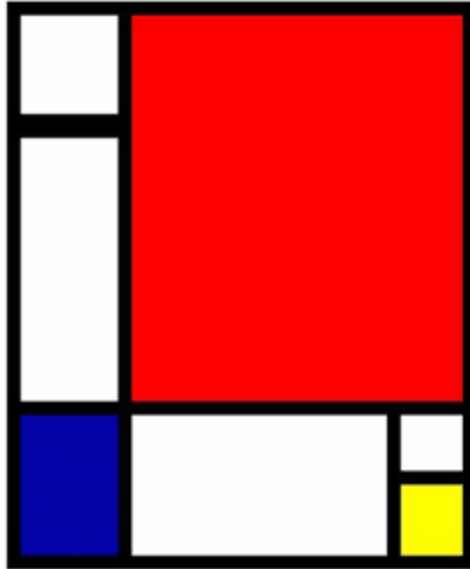
The Vitruvian Man ([nowarchy.blogspot.com/2009/05/vitruvian-man-real-davinci-code.html](http://nowarchy.blogspot.com/2009/05/vitruvian-man-real-davinci-code.html))

In the Spanish/Catalan surrealist painter Salvador Dalí's (1904 – 1989) *The Last Supper*, the dimensions of the canvas are those of a golden rectangle. The huge dodecahedron that is suspended above and behind Jesus has edges that are in golden ratio to one another.



The Last Supper ([piccolo.rispostesenzadomanda.com/post/117427563/hessianonulo-bluesiren-the-last](http://piccolo.rispostesenzadomanda.com/post/117427563/hessianonulo-bluesiren-the-last))

The Dutch Neo-Plasticist painter Piet Mondrian (1872 – 1944) used the golden section extensively in his geometrical paintings. For example, this is clearly seen in his *Composition in Red, Yellow, and Blue*:



Piet Mondrian, *Composition in Red, Yellow, and Blue*  
([http://fotogenetic.dearingfilm.com/golden\\_rectangle.html](http://fotogenetic.dearingfilm.com/golden_rectangle.html))

The Swiss French architect and designer Le Corbusier (1887 – 1965) centered his design philosophy on systems of harmony and proportion and a mathematical order of the universe, and his buildings were closely bound to the golden ratio and the Fibonacci series.

Fibonacci numbers and the golden ratio can also be found in music. The number of measures in certain sections of works of Johann Sebastian Bach (1685 - 1750) and Frédéric Chopin (1810 – 1849) are based on the golden ratio. In the Hungarian composer Béla Bartók's (1881 – 1945) *Music for Strings, Percussion, and Celesta*, the xylophone progression occurs at the intervals given by the Fibonacci numbers. In Claude Debussy's (1862 – 1918) *Reflets dans l'Eau* (Reflections on Water), the golden ratio is used to organize the sections in the music.

Or take Georg Cantor's (1845-1918) theory of infinity and transfinite numbers. As David Hilbert said, "No other question has ever moved so profoundly the spirit of man" as the quest to understand infinity. Many paradoxes arose even when the most accomplished mathematicians tried to work with this concept. Galileo (1564 – 1642) aptly noted that these paradoxes were a result of our attempting "...with our finite minds, to discuss the infinite, assigning to it those properties which we give to the finite and limited..." - like claiming that the whole is always strictly greater than its parts.

Prior to Cantor's work, the concept of set was a rather elementary one that had remained relatively unchanged from the time of Greek mathematicians and reflected nothing more than the antiquated ideas of Aristotle (384 B.C.E. – 322 B.C.E.). There were only finite sets, for infinite sets were considered to belong more to the realm of philosophy than that of mathematics.

Cantor started out with the simple idea of *bijection* in set theory. A *bijection* is a function  $f$  from a set  $X$  to a set  $Y$ , such that, for every  $y$  in  $Y$ , there is exactly one  $x$  in  $X$  with the property that  $f(x) = y$  (that is,  $f$  is an *injection*), and no unmapped element exists in  $Y$  (that is,  $f$  is also a *surjection*).

He used this concept to “count” the number of elements in a set, and eventually, to demarcate the difference between finite and infinite sets. The *cardinality* of a finite set  $X$ ,  $\text{card}(X)$ , is the number of elements of the set. For example, if  $X = \{5, 10, 15, 20\}$ , then  $\text{card}(X) = 4$ . Thus,  $\text{card}(X) = n$  means there exists a bijection between  $\{1, 2, 3, \dots, n\}$  and  $X$ . Infinite sets were those sets for which no bijection to a finite set could be found. Infinite sets were still divided into two types: *denumerable* (or *countably infinite*), sets from which a bijection to the set of natural numbers existed, and *nondenumerable*, those that could not be put in a one-to-one correspondence with the set of natural numbers.

Cantor then introduced the concept of the *power set* of a set  $A$ , the set of all possible subsets of  $A$ , and showed that no set can be put into one-to-one correspondence with its power set (Cantor’s Theorem).

This ended up establishing a very bizarre conclusion (Cantor’s Theorem): the set of real numbers was “more numerous” (had greater power - *mächtigkeit*) than the set of integers. So if, following Cantor, we denote the cardinality of integers (or of the natural numbers or of the rational numbers) by  $\aleph_0$  (aleph-null) and the cardinality of the set of real numbers by  $c$ , we have  $\aleph_0 < c$ .

In fact, Cantor’s Theorem implied that there was a well-ordered infinite set of cardinal numbers

$$1 < 2 < \dots < n \dots < \aleph_0 < c < \dots$$

i.e., there were infinitely many infinities.

Cantor's theory, which is now a well-established part of mathematics, encountered intense antagonism from his contemporaries, such as Leopold Krönecker (1823 – 1891), who called him a “charlatan,” and Jules Henri Poincaré (1854 -1912). Some theologians saw Cantor’s work as a challenge to absolute infinity, i.e., to the true nature of God. Indeed, Cantor’s recurring attacks of depression have been blamed on such hostile attitudes. Even death did not fully absolve him. The twentieth century philosopher Ludwig Wittgenstein (1889 – 1951) referred to Cantor’s theory of infinity as “utter nonsense” and “laughable.”

There is a very interesting question related to Cantor’s theory, namely, *the continuum hypothesis*. It asserts that there is no set whose power is greater than that of the integers and less than that of the real numbers; that is, there is no cardinal between  $\aleph_0$  and  $c$ . Cantor tried, in vain, for many years to prove it. Later, David Hilbert proposed this as the first problem of his Twenty-three Open Problems in his famous 1900 address at the International Congress of Mathematicians in Paris. A 1940 theorem of Kurt Gödel (1906 – 1978) and a 1963 theorem of Paul Cohen (1934 – 2007) together imply that the continuum hypothesis can neither be proved nor disproved using standard set theory axioms. So there are mathematical statements that can neither be proved nor disproved.

I wanted to be able to communicate these superb and brilliant ideas and theories to others the best way I could. I wanted to find better ways of exposing my students to problems and questions rather than to already complete pieces of information and simple answers. I wanted them to appreciate that the role played by imagination was equally important in mathematics as the role played by proofs and deductive reasoning. I wanted, to paraphrase Albert Einstein (1879 – 1955), “to awaken joy in creative expression



and knowledge". Thus, I decided to obtain a degree in Mathematics Education as well. I started the doctoral program in Mathematics Education at the Department of Mathematics and Statistics of American University in 2003. I completed my studies in May 2008. My dissertation was titled *Study of Instantaneous Rate of Change in a Historical Context*.

This study was essentially comprised of four parts. The primary purpose of the first part was to analyze the historical development of the concept of instantaneous rate of change, determine, in particular, the reasons for its comparatively late induction into the realm of natural sciences, and explore the uniquely significant role it has played in transforming mathematics as well as physics into their modern forms. The second part was devoted to the development of various learning models as applied to mathematics. In the third part I analyzed certain issues concerning foundations of mathematics and their impact on mathematics education. Finally, I expounded on how a well-designed assortment of the historic development of the notion of instantaneous rate of change, a suitable pedagogical model, and a correct philosophical foundational approach would elucidate certain fundamental concepts in geometry, algebra, and calculus; enrich students' understanding and appreciation of mathematics; and alter the common misperception that mathematics is merely a list of facts, by engendering a viable alternative to the usual prosaic teaching styles associated with these topics.

### A Short Reading List

Any serious student of mathematical sciences should take to heart Sir Isaac Newton's (1642 – 1726) modest motto: "If I have seen further it is only by standing on the shoulders of giants," and read the original works of giants.

At minimum, this reading list should start with *Elements*, the monumental work of Euclid of Alexandria (c. 300 B.C.E. - ?)



Title page of Sir Henry Billingsley's (? – 1606) first English version of Euclid's Elements, 1570

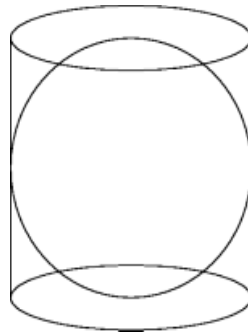
In this book, based on five postulates

1. Any two points can be connected by one and only one straight line segment

2. Any line segment can be extended to a line
3. Given any point and any line segment starting at that point, there is a circle with that point as its center and that segment as its radius
4. All right angles are equal
5. Given a line  $L$  and a point  $P$  not on  $L$ , there is one and only one line through  $P$  that never meets  $L$ .

Euclid was able to establish a geometry, that was the . (?)

By reading excerpts of Archimedes's (c. 287 B.C.E. – c. 212 B.C.E.) *Method*, the student should be able to understand what made him request the following figure to be inscribed on his tombstone



Archimedes's Tombstone - a sphere inscribed in a cylinder

Archimedes had computed the ratio of their volumes, and this was one of his favorite results. The computation goes as follows: if the radius of the sphere is  $r$ , the radius of the cylinder would also be  $r$  and its height,  $2r$ . Thus, the volume of the sphere would be

$$V_s = \frac{4}{3}\pi r^3$$

and the volume of the cylinder would be

$$V_c = \pi r^2(2r) = 2\pi r^3$$

Hence,

$$\frac{V_s}{V_c} = \frac{2}{3}$$

the ratio Archimedes was so proud of.

One mathematician that fascinated me the most was Sir Isaac Newton. Before Newton, ***argumentum ad verecundiam*** (*argument to respect*), that is, an argument of the type

1. Source  $A$  says that statement  $p$  is true.
2. Source  $A$  is authoritative.
3. Therefore, statement  $p$  is true.

was a very common practice in the sciences, and in most cases the authority was Aristotle. Newton, the true modernist he was, had indicated his dissatisfaction with this methodology in his *Quaestiones Quaedam Philosophicae* [Certain Philosophical Questions] (c. 1664)

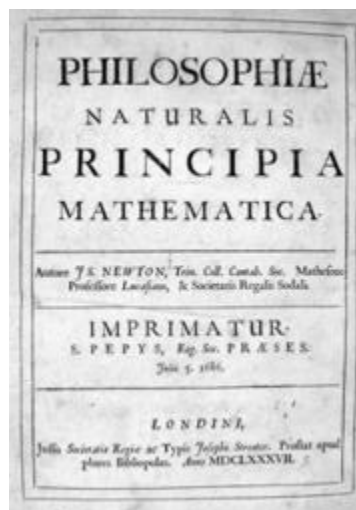
*Amicus Plato, amicus Aristoteles, magis amica veritas*

(Plato is my friend, Aristotle is my friend, but my greatest friend is truth).

That is why I feel Newton's *Philosophiæ Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy), a work in three volumes, where modern day calculus and classical mechanics were established, should be in everyone's reading list.

*Principia* was first published in July of 1687. Within the next 40 years Newton published two further editions, concluding the last edition by the now famous expression "*Hypotheses non fingo*" (I feign no hypotheses).

In this book, besides many other novel results, Newton stated his Laws of Motion, thus forming the foundation of classical mechanics, established his Law of Universal Gravitation, and derived Kepler's laws of planetary motion, which, up to that time, had existed only on an empirical level.



Title page of *Philosophiæ Naturalis Principia Mathematica* (First edition - 1687)

Although the precise language of modern calculus was largely absent from the *Principia* - Newton gave most of his proofs using limits of ratios of vanishing small geometric quantities - it is still considered to be one of the most important works in the history of sciences. The French mathematician Alexis Clairaut (1713 – 1765) in his paper *Du systeme du monde, dans les principes de la gravitation universelle*, which was published in *Histoires et Memoires de l'Academie Royale des Sciences* in 1749, wrote that *Principia*

... spread the light of mathematics on a science [physics] which up to then had remained in the darkness of conjectures and hypotheses.

It is easy to miss the dynamic and humanistic nature of mathematical sciences and to reduce them to a series of definitions, axioms, and theorems devoid of any historic or cultural context, often promulgated by rather diffident introverts. To those who may claim mathematics is static or that mathematics has not undergone any revolutionary changes like the other sciences, I counter with non-Euclidean geometries. All of these geometries are based on the first four of Euclid's postulates, but each uses its own version of the fifth postulate. In one case, there are no lines parallel to a given line from a point not on the line, and in the other case, there are infinitely many. In 1868 the Italian mathematician Eugenio Beltrami (1835 – 1900) proved that non-Euclidean geometries were as logically consistent as their Euclidean counterpart.

To expose the inherent limitations of the thought system that claims all of mathematics can be studied through an axiomatic approach, I offer *The Incompleteness Theorems*, proven by Kurt Gödel in 1931. They state that any consistent system of axioms is incapable of proving certain truths about arithmetic. This result has shown that Hilbert's program of finding a complete and consistent set of axioms for all of mathematics is impossible.

To those who claim problems that sound simple must have simple solutions, I will give them three problems and actually help with the solutions of the first two. For the third, they are on their own.

1. Why is a writer's life complex?
2. Which type of numbers are best for you health?
3. Show that any even number can be written as a sum of two prime numbers. For example,  $20 = 13 + 7$ ,  $44 = 31 + 13$ .

This question, known as the *Goldbach conjecture*, was proposed in 1742 by the Prussian mathematician Christian Goldbach (1690 – 1764) in a letter to Euler

haben, nicht besitzen, ob man aber schon mal gefunden hat,  
 & wenn nicht jedes kleine number muss modo in die quadrate  
 divisibiles geben auf solche Weise will ich auch eine conjecture  
 hazardieren: dass jede Zahl welche sich zerlegen lässt in primis  
 zusammengesetzt ist ein aggregatum so vieler numerorum  
 primorum, als man will; in unitatem mit der 1 zusammen  
 dass auf die conjectur omnia unitatem 2. zum Beispiel  
 $4 = \begin{cases} 1+1+1+1 \\ 1+1+2 \\ 1+3 \end{cases}$   $5 = \begin{cases} 1+1+1+1+1 \\ 1+1+1+2 \\ 1+1+1+1+1 \end{cases}$   $6 = \begin{cases} 1+1+1+1+1+1 \\ 1+1+1+2+1 \\ 1+1+1+1+2 \\ 1+1+1+1+1+1 \end{cases}$  &c.  
 Darauf folgen ein paar observationes so demonstrirt unter  
 dem Pennant:  
 Si  $v$  sit functio ipsius  $x$  cuiusmodi ut facta  $v = c$  numero cui-  
 cusque, determinari possit  $x$  per  $c$  et reliquas constantes in functi-  
 one expressas, poterit etiam determinari valor ipsius  $x$  in aen-  
 quatione  $v^{n+1} = (2v+1)(v+1)^{n-1}$  |  $\frac{v^{n+1} - (2v+1)(v+1)^{n-1}}{(2v+1)(v+1)^{n-1}}$  dass  $v=0$   
 Si incipiat curvas cuius abscissa sit  $x$  applicata deus sit  
 summa seriei  $\frac{x^n}{n \cdot 2^n}$  posita  $x$  pro exponente terminorum, hoc est,  
 applicata  $= \frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 2^2} + \frac{x^3}{3 \cdot 2^3} + \frac{x^4}{4 \cdot 2^4} + \text{etc.}$  dico, si fuerit  
 abscissa  $= 1$ , applicatum fore  $= \frac{1}{2} = \frac{1}{2}$  |  $\frac{1}{2}$  hoc est  $\frac{1}{2}$  und  $\frac{1}{2} = \frac{1}{2}$   
 $2 \dots \dots \dots 2 \cdot 2$   
 $3 \dots \dots \dots 3 \cdot 2$   
 $4 \text{ vel major} \dots \dots \dots \text{infinitum}$   
 Ich verfahe mit aller ansehnlichkeit Beschreibung  
 L'Amour Hochachtungsvoll  
 Moskau 7. Jun. st. 12. 1742. J.  
 Goldbach

Goldbach's Letter to Euler  
 ([http://upload.wikimedia.org/wikipedia/commons/thumb/1/18/Letter\\_Goldbach-Euler.jpg/400px.1](http://upload.wikimedia.org/wikipedia/commons/thumb/1/18/Letter_Goldbach-Euler.jpg/400px.1))

Answers

1. Because it has both real and imaginary parts.
2. Natural numbers.

And finally to those who harbor a bit of a sense of humor, I have some good news - they can prove theorems as utterly expedient as the following one:



**Theorem.** *Death comes to no man.*

*Proof.* As is well known, when we approach death our whole life flashes in front of us. In order to be complete, this synopsis must also include the moment we approached death and the flashback of our life. But then there has to be another flashback of our life, including the moment we had the previous flashback, and so on, *ad infinitum*. Hence, although we may approach death, we will never actually reach it (*Leinbach's Proof*).